Antwoorden Collatz

Opgave 1

C(1) = 4, C(2) = 1, C(3) = 10, C(4) = 2, C(5) = 16, C(6) = 3, C(7) = 22, C(8) = 4, C(9) = 28, C(10) = 5

Opgave 2a

Startgetal 1: 1
Startgetal 2: 2,1
Startgetal 3: 3, 10, 5, 16, 8, 4, 2, 1
Startgetal 4: 4, 2, 1
Startgetal 5: 5, 16, 8, 4, 2, 1
Startgetal 6: 6, 3, 10, 5, 16, 8, 4, 2, 1
Startgetal 7: 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
Startgetal 8: 8, 4, 2, 1
Startgetal 9: 9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
Startgetal 10: 10, 5, 16, 8, 4, 2, 1
Startgetal 11: 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
Startgetal 12: 12, 6, 3, 10, 5, 16, 8, 4, 2, 1
Startgetal 13: 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
Startgetal 14: 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
Startgetal 15: 15, 46, 23, 70, 35, 106, 53, 170, 85, 256, 128, 64, 32, 16, 8, 4, 2, 1
Startgetal 16: 16, 8, 4, 2, 1
Startgetal 17: 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
Startgetal 18: 18, 9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
Startgetal 19: 19, 58, 29, 88, 44, 22, 11, 3, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
Startgetal 20: 20, 10, 5, 16, 8, 4, 2, 1

Opgave 2b:
Startgetal 27: 27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1

Opgave 3:

Startgetal 1: 1, 2, 1, 2, 1, …..
Startgetal 2: 2, 1, 2, 1, 2, …..
Startgetal 3: 3, 8, 4, 2, 1, 2, 1, …..
Startgetal 4: 4, 2, 1, 2, 1, 2, 1, …..
Startgetal 5: 5, 14, 7, 20, 10, 5, 14, 7, 20, 10, 5, …..
Startgetal 6: 6, 3, 8, 4, 2, 1, 2, 1, …..
Startgetal 7: 7, 20, 10, 5, 14, 7, 20, 10, 5, 14, 7, 20, 10, 5, …..
Startgetal 8: 8, 4, 2, 1, 2, 1, 2, 1, …..
Startgetal 9: 9, 26, 13, 38, 19, 56, 28, 14, 7, 20, 10, 5, 14, 7, 20, 10, 5, 14, 7, 20, 10, 5, …..
Startgetal 10: 10, 5, 14, 7, 20, 10, 5, 14, 7, 20, 10, 5, …..
Hypothese 1 geldt niet.

Opgave 4:

Startgetal 1: 1, 6, 3, 16, 8, 4, 2, 1, 6, …..
Startgetal 2: 2, 1, 6, 3, 16, 8, 4, 2, 1, 6, …..
Startgetal 3: 3, 16, 8, 4, 2, 1, 6, 3, 16, 8, 4, 2, 1, 6, …..
Startgetal 4: 4, 2, 1, 6, 3, 16, 8, 4, 2, 1, 6, …..
Startgetal 5: 5, 26, 13, 66, 33, 166, 83, 416, 208, 104, 52, 26, 13, 66, 33, 166, 83, 416, 208, 104, 52, 26, 13, …..
Startgetal 6: 6, 3, 16, 8, 4, 2, 1, 6, 3, 16, 8, 4, 2, 1, 6, …..
Startgetal 7: 7, 36, 18, 9, 46, 23, 116, 58, 29, 146, 73, 366, 183, 916, 458, 229, 1146, 572, 286, 143, 716, 358, 179, 896, 448, 224, 112, 56, 28, 14, 7, 36, 18, …..
Startgetal 8: 8, 4, 2, 1, 6, 3, 16, 8, 4, 2, 1, 6, …..
Startgetal 9: 9, 46, 23, 116, 58, 29, 146, 74, 37, 186, 93, 466, 233, 700, 350, 175, 876, 438, 219, 1096, 548, 275, 137, 686, 343, 1716, 858, 429, 2196, 1098, 549, 2746, 1373, ?
Startgetal 10: 10, 5, 26, 13, 66, 33, 166, 83, 416, 208, 104, 52, 26, 13, 66, 33, 166, 83, 416, 208, 104, 52, 26, 13, …..
Hypothese 1 geldt niet.

Opgave 5:

Startgetal 1: 1, 1, 1, …..
Startgetal 2: 2, 3, 2, 3, 2, 3, …..
Startgetal 3: 3, 2, 3, 2, 3, …..
Startgetal 4: 4, 5, 7, 9, 6, 4, 5, 7, 9, 6, 4, …..
Startgetal 5: 5, 7, 9, 6, 4, 5, 7, 9, 6, 4, …..
Startgetal 6: 6, 4, 5, 7, 9, 6, 4, …..
Startgetal 7: 7, 9, 6, 4, 5, 7, 9, 6, 4, …..
Startgetal 8: 8, 11, 15, 10, 13, 17, 23, 31, 41, 55, 73, ?
Hypothese 1 geldt niet.

Opgave 6:

Startgetal 1: 1, 1, 1, 1, 1, …..
Startgetal 2: 2, 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 2, 7, …..
Startgetal 3: 3, 10, 5, 16, 2, 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 2, 7, …..
Startgetal 4: 4, 2, 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 2, 7, …..
Startgetal 5: 5, 16, 2, 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 2, 7, …..
Startgetal 6: 6, 3, 10, 5, 16, 2, 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 2, 7, …..
Startgetal 7: 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 2, 7, …..
Startgetal 8: 8, 2, 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 2, 7, …..
Startgetal 9: 9, 3, 10, 5, 16, 2, 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 2, 7, …..
Startgetal 10: 10, 5, 16, 2, 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 2, 7, …..
Hypothese 1 geldt niet.

Opgave 7a:

0, 0, 0, …..

Opgave 7b:

Startgetal -1: -1, -2, -1, -2, -1, …..
Startgetal -2: -2, -1, , -2, -1, -2, -1, …..
Startgetal -3: -3, -8, -4, -2, -1, -2, -1, -2, -1, …..
Startgetal -4: -4, -2, -1, -2, -1, -2, -1, …..
Startgetal -5: -5, -14, -7, -20, -10, -5, , -14, -7, -20, -10, -5, …..

Opgave 7c:

Hypothese 1 is niet waar.

Opgave 7d:

Startgetal -1: -1, -4, -2, -1, -4, -2, -1, …..
Startgetal -2: -2, -1, -4, -2, -1, -4, -2, -1, …..
Startgetal -3: -3, -10, -5, -16, -8, -4, -2, -1, …..
Startgetal -4: -4, -2, -1, …..
Startgetal -5: -5, , -16, -8, -4, -2, -1, …..

Opgave 7e:

Hypothese: De rij die ontstaat door de functie D herhaaldelijk toe te passen op een negatief geheel startgetal komt altijd uit op -1.

Opgave 8a:

T(1) = 0, T(2) = 1, T(3) = 7, T(4) = 2, T(5) = 5, T(6) = 8, T(7) = 16, T(8) = 3, T(9) = 19, T(10) = 6, T(11) = 14, T(12) = 9, T(13) = 9, T(14) = 17, T(15) = 17, T(16) = 4, T(17) = 12, T(18) = 20, T(19) = 20, T(20) = 7

Opgave 8b:

T(27) = 111

Opgave 9a:

Startgetal 16n + 2: 16n + 2, 8n + 1, 24n + 4, 12n + 2, 6n + 1, 18n + 4
Startgetal 16n + 3: 16n + 3, 48n + 10, 24n + 5, 72n + 16, 36n + 8, 18n + 4
Beide startgetallen komen na 5 stappen uit op 18n + 4, dus geldt: T(16n + 2) = T(16n + 3).

Opgave 9b:

Startgetal 32n – 10: 32n – 10, 16n – 5, 48n – 14, 24n – 7, 72n – 20, 36n – 10, 18n – 5, 54n – 14
Startgetal 32n – 9: 32n – 9, 96n – 26, 48n – 13, 144n – 38, 72n – 19, 216n – 56, 108n – 28, 54n – 14
Beide startgetallen komen na 7 stappen uit op 54n – 14, dus geldt: T(32n – 10) = T(32n – 9).

Opgave 10a:

Startgetal 32n + 2: 32n + 2, 16n + 1, 48n + 4, 24n + 2, 12n + 1, 36n + 4
Startgetal 32n + 3: 32n + 3, 96n + 10, 48n + 5, 144n + 16, 72n + 8, 36n + 4
Beide startgetallen komen na 5 stappen uit op 36n + 4, dus geldt: T(32n + 2) = T(32n +3)

Opgave 10b:

Startgetal 32n + 4: 32n + 4, 16n + 2, 8n + 1, 24n + 4, 12n + 2, 6n + 1, 18n + 4
Startgetal 32n + 5: 32n + 5, 96n + 16, 48n + 8, 24n + 4, 12n + 2, 6n + 1, 18n + 4
Startgetal 32n + 6: 32n + 6, 16n + 3, 48n + 10, 24n + 5, 72n + 16, 36n + 8, 18n + 4
De drie startgetallen komen na 6 stappen allemaal uit op 18n + 4, dus geldt:
T(32n + 4) = T(32n + 5) = T(32n + 6).

Opgave 11a:
Startgetal 2^n: 2^n, 2^{n-1}, ……., $2^0 = 1$ Van n-1 tot 0 zijn n stappen, dus $T(2^n) = n$.
Formeel is dit een bewijs met Volledige Inductie.

Opgave 11b:
Na n keer door 2 delen komt de Collatzrij uit op m. Vanaf m is het nog T(m) stappen tot 1.
Zolang hypothese 1 niet bewezen is, zou T(m) = ∞ kunnen zijn, maar dan is $T(2^n.m)$ ook ∞.

Opgave 12:

Collatzrij voor 8m – 1: 8m – 1, 24m – 2, 12m – 1, 36m – 2, 18m – 1, 54m – 2, 27m – 1.
Of 27m – 1 even of oneven is, hangt van m af. Verder geldt:
24m – 2, 12m – 1, 36m – 2, 18m – 1, 54m – 2, 27m – 1 zijn alle groter dan 8m – 1.

Opgave 13:

Verloopsinductie naar n.

Basis: n = 1 is vanzelfsprekend.

Stel de hypothese is waar voor de startgetallen 1 …. n.
Inductiestap:

Je moet drie gevallen onderscheiden n + 1 = 2k, n + 1 = 4k +1 en n + 1 = 4k – 1

1. n + 1 = 2k
   pas eenmaal M toe: M(n + 1) = M(2k) = k.
   k < 2k = n + 1. De inductiehypothese geldt voor k.
2. n + 1 = 4k + 1
   pas M driemaal toe: 12k + 4, 6k + 2, 3k +1
   3k + 1 < 4k + 1. De inductiehypothese geldt voor 3k + 1
3. n + 1 = 4k -1
   pas M driemaal toe: 12k – 4, 6k – 2, 3k – 1
   3k – 1 < 4k – 1. De inductiehypothese geldt voor 3k – 1.

Opgave 14a:

Niveau 9: 512, 85, 84, 80, 13, 12

Niveau 10: 1024, 170, 168, 160, 26, 24

Niveau 11: 2048, 341, 340, 336, 320, 53, 52, 48

Niveau 12: 4096, 682, 680, 113, 672, 640, 106, 104, 17, 96

Niveau 13: 8192, 1365, 1364, 227, 1360, 226, 1344, 1280, 213, 212, 35, 208, 34, 192

Niveau 14: 16384, 2730, 2728, 454, 2720, 453, 452, 75, 2688, 2560, 426, 424, 70, 416, 69, 68, 11, 384

Niveau 15: 32768, 5461, 5460, 5456, 909, 908, 151, 5440, 906, 904, 150, 5376, 5120, 853, 852, 848, 141, 140, 23, 832, 138, 22, 768

Niveau 16: 65536, 10920, 10912, 1818, 1816, 302, 10880, 1813, 1812, 1808, 301, 300, 10752, 10240, 1706, 1704, 1696, 282, 280, 46, 1664, 277, 276, 272, 45, 44, 7, 1536

Niveau 17: 131072, 21845, 21840, 21824, 3636, 3632, 605, 604, 21760, 3626, 3624, 3616, 602, 600, 21504, 20480, 3413, 3412, 3408, 3392, 565, 564, 560, 93, 92, 15, 3328, 554, 552, 544, 90, 88, 14

Niveau 18: 262144, 43690, 43680, 43648, 7272, 7264, 1210, 1208, 201, 43520, 7253, 7252, 7248, 7232, 1205, 1204, 1200, 43008, 40960, 6826, 6824, 1137, 6816, 6784, 1130, 1128, 1120, 186, 184, 30, 6656, 1109, 1108, 1104, 1088, 181, 180, 176, 29, 28, 6144

Niveau 19: 524288, 87381, 87380, 14563, 87360, 87296, 14549, 14544, 14528, 2421, 2420, 403, 2416, 402, 87040, 14506, 14504, 24,17, 14496, 14464, 2410, 2408, 401, 2400, 86016, 81920, 13653, 13652, 2275, 13648, 2274, 13632, 13568, 2261, 2260, 2256, 2240, 373, 372, 368, 61, 60, 13312, 2218, 2216, 369, 2208, 2176, 362, 360, 352, 58, 56, 9, 12288

Opgave 14:

Op niveau 111.

Opgave 15a: Merk op dat het aantal getallen in de Collatzrij één hoger is dan het niveau waarop het getal in de boom staat. Dus de getallen met een Collatzrij met lengte 9 staan in de boom op niveau 8, dus het aantal is 4.

Opgave 15b: 28.

Opgave 16a: Op niveau n staat het getal: 21.2^{n-7} met $n \geq 7$

Opgave 16b: Op niveau n staat het getal 27.2^{n-111} met $n \geq 111$

Opgave 16c: 82

Opgave 16d: 41

Opgave 17a: 2^{741}

Opgave 17b: $(2^{740} - 1)/3$